The effectiveness of ineffective face masks for reducing the risk of COVID-19 to the wearer

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1 Introduction

Many measures have been enacted to try to control the spread of the SARS-CoV-2 virus and reduce the incidence of COVID-19. These have varied from country to country and over time, and include lockdowns, shelter-in-place orders, prohibition of large gatherings, closure of restaurants, bars, and non-essential shops, recommendations for social distancing and hand-washing, not touching the face, and disinfecting of surfaces.

Within these measures, the use of masks has been controversial and the recommendations contradictory and poorly communicated. Early in the pandemic, surgical quality masks were in short supply and health care workers in many places were unable to obtain them, putting those workers at risk because they work in close contact with COVID-19 patients. Agencies, including RIVM in the Netherlands and CDC in the United States downplayed the value of the wearing of masks by the general public in order to make masks more available to health workers.

Over time, that narrative changed to the claim that wearing of even simple homemade masks would help to prevent the spread of SARS-CoV-2 *from* asymptomatic wearers of the mask to susceptible individuals. But agencies kept repeating the claim that such masks would not reduce transmission *to* the wearer of the mask. Arguments for this position tended to cite figures on the fraction of infectious particles that could pass through a cloth mask, and contrast this with the protection afforded by high quality masks (N95) appropriately reserved for medical staff. I have seen figures quoted to the effect that 40–80% of particles pass through such a mask. Which implies that 20–60% of the particles do not pass. In an experimental study at RIVM, van der Sande et al. (2008) found that simple homemade cloth masks reduced inward transmission by 66%; see Huang (2020) for a recent survey of information on the mechanics and the biology of COVID-19 transmission.

The claim that masks provide no protection to the wearer is challenged by noting that even if a mask does not stop all infectious particles, stopping some particles provides some degree of protection. It is this issue that I want to address here. How effective can an ineffective mask be at reducing infection risk for the wearer? The answer is that it can be more effective than most people believe.

Nothing here should be taken to downplay the value of other ways to reduce risk, including hand-washing, social distancing, self-quarantine when ill, and the value of masks at reducing infection from asymptomatic but infected mask wearers.

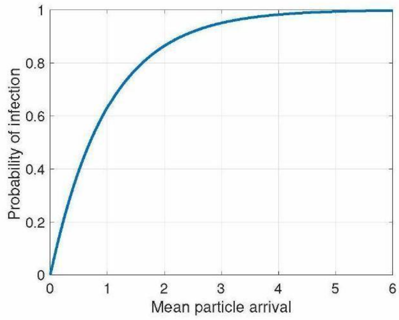

Figure 1: The probability of infection as a function of the rate of arrival of infections particles.

2 Reducing the mean, reducing the probability

Consider infection resulting from the arrival of infectious particles to the nose/mouth, where they are inhaled, thereby giving the SARS-CoV-2 virus access to the lungs. SARS-CoV-2 is transmitted in droplets expelled by breathing, talking, sneezing, and coughing. Ignoring the complicated dynamics of these particles, consider the mean number of infectious particles arriving, per unit time, to the nose/mouth.

. The probability of infection per unit time is the probability of receiving at least one of these particles, which is given by

$$p = 1 - e^{-m}. \tag{1}$$

(Figure 1). If no particles arrive, the probabilty of infection is 0. If the number of particles arriving becomes extremely large, the probability of infection goes to 1.

The benefit of a mask to the wearer is its reduction of the rate of arrival of infectious units to the nose/mouth, i.e., reducing m. The effect of this reduction is given by the elasticity of p with respect to m; that is, the proportional change in p that results from a proportional change in m. This elasticithy is calculated from the derivative of infection probability to a change in m,

$$\frac{dp}{dm} = e^{-m} \tag{2}$$

The elasticity of p with respect to m is

$$\frac{\varepsilon p}{\varepsilon m} = \frac{m}{p}\frac{dp}{dm} \tag{3}$$

$$= \frac{me^{-m}}{1-e^{-m}} \tag{4}$$

The limiting cases $m \to 0$ (a low-virus environment) and $m \to \infty$ (a high-virus environment) are of particular interest to us. Using l'Hopital's rule gives

$$\lim_{m\to 0} \frac{\varepsilon p}{\varepsilon m} = 1 \tag{5}$$

and

$$\lim_{m\to\infty} \frac{\varepsilon p}{\varepsilon m} = 0 \tag{6}$$

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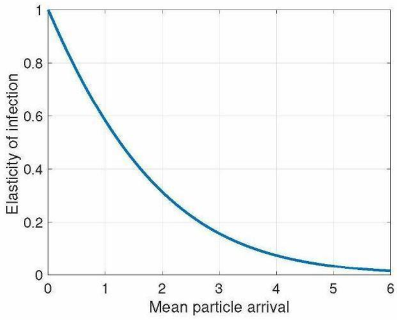

Figure 2: The elasticity of the probability of infection with respect to the mean rate m of arrival of particles, as a function of m.

Implications. An elasticity of 1 implies that a specified percent reduction in m translates directly into the same percent reduction in p. That is, if an ineffective cloth mask blocks 10% of the infectious particles arriving at the nose/mouth, it reduces the risk of infection by 10%. If the cloth mask blocks 50% of the particles, it reduces the risk of infection by 50%.

That is, in the low-virus environments in which most people spend most of their time, an 'ineffective' cloth mask actually provides an appreciable reduction of the risk of infection.

In high-virus environments, such as those confronted by health care workers and hospital staff, reducing the risk of infection appreciably requires a very large reduction in m. This can be achieved only by a mask that blocks almost all particles. This reinforces the importance of making sure that health care workers have access to highly effective medical-grade masks (and other PPE).

3 Notes

1. The infection probability in equation (1) follows directly from describing the arrival of particles as a Poisson process. I suspect that the qualitative conclusions of the analysis hold for any infection probability function that is a continuous, concave function of m with the properties that $p(0) = 0$ and $\lim_{m\to\infty} p(m) = 1$. These properties are reasonable: if there are no particles, there is no probability of infection; if one is swimming in a sea of particles, infection is certain.

2. Droplets produced by breathing, talking, singing, sneezing, coughing, etc. are produced in a range of sizes with a range of velocities, affected by gravity and evaporation in complicated ways. To treat these as simply infectious units is an obvious simplification.

3. This analysis obviously says nothing about infection arriving by routes other than inhalation.

4. These calculations obviously do not include interactions with other health behaviors related to SARS-CoV-2 , e.g., would overconfidence in mask efficiency lead to failure to respect social distancing.

5. The received wisdom that masks serve only to keep asymptomatic carriers from infecting others results from two errors. The first is confusion of the needs of masks for health care workers and masks for the public. The fact that ineffective masks would not

suffice for health workers exposed repeatedly and for long periods to sick patients says nothing about what they would do for people in ordinary circumstances. The second is the failure to recognize that all risks are relative. The fact that an ineffective mask does not reduce the risk of infection to zero says nothing about how much reduction it does provide, and about whether that reduction of risk is appreciable.

6. Risk reduction is cumulative. The risk of death from automobile accidents is reduced by seat belts, airbags, equipment regulations, speed limits, and so on. None of those features reduce the risk to zero, but in combination they reduce risk significantly. The reduction of infection probability by wearing a mask in low-virus environments is an addition to, not a replacement for social distancing, hand-washing, avoiding crowds and unnecessary travel, etc.

7. How consequential are the reductions in infection probability that might be produced? A comparison may make it more understandable. How much benefit do healthy lifestyle choices provide? Such benefits are customarily presented as hazard ratios, which measure the proportional increases or reductions in rates of some outcome, relative to a baseline. A typical example can be found in Li et al. (2018), who analyze the effects of healthy lifestyle factors on mortality due to cardiovascular disease. They report that a high score on a healthy eating index reduces the risk by 33%, a high level of physical activity by 61%, a moderate compared to a high level of alcohol consumption by 15%, a normal BMI compared to mild obesity by 40%. None of these are perfectly effective in preventing death from cardiovascular disease. Yet, we have no problem believing that the partial effectiveness of healthy eating, exercise, and body weight are worthwhile. These hazard ratios are comparable to the reductions in infection probabilty from an ineffective face mask that, in a low-virus environment, blocks 15–60% of infectious particles.

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